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**A human rights-consistent approach to
multidimensional welfare measurement applied
to sub-Saharan Africa**

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Abstract: The rights-based approach to development targets progress towards the realization of 30 articles set forth in the Universal Declaration of Human Rights. Progress is frequently measured using the multidimensional poverty index. While elegant and useful, the multidimensional poverty index is in reality inconsistent with the Universal Declaration of Human Rights principles of indivisibility, inalienability, and equality. We show that a first-order dominance methodology maintains consistency with basic principles, discuss the properties of the multidimensional poverty index and first-order dominance, and apply the measures to 26 African countries. We conclude that the multidimensional poverty index and first-order dominance are useful complements that should be employed in tandem.

Keywords: first-order dominance, multidimensional welfare measurement, sub-Saharan Africa, Demographic and Health Survey, human rights-based approach

JEL classification: C81, D39, D63, I3, O12, O55

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1 Introduction

On 10 December 1948, the United Nations General Assembly (UNGA) adopted the Universal Declaration of Human Rights (UDHR) as ‘a common standard of achievement for all peoples and all nations’. The UDHR contains 30 articles that define a series of fundamental human rights. The large majority of these articles refer to laws or norms that govern the functioning of society. For example, article 2 states that the rights and freedoms set forth in the UDHR apply without distinction of any kind, such as race, religion, or gender (sex); and article 15 affirms that everyone has a right to a nationality. These and most other articles within the Declaration can in principle be followed without reference to the material circumstance faced by the society in question. However, a smaller subset of the articles is notably easier for wealthier societies to fulfil than for poorer societies. For instance, articles 25 and 26 assert, respectively, that everyone has the right to an adequate standard of living and that everyone has the right to education, particularly at elementary levels.

Poor countries may confront enormous or even insurmountable (within short time frames) obstacles in meeting these obligations of the UDHR, regardless of the collective desire of those with decision-making power within those societies (labelled duty-bearers) to fulfil them. This gap between stated goals and circumstances on the ground makes it necessary to interpret the UDHR as an aspirational declaration whose tenets, under favourable circumstances, gradually become satisfied for an increasing share of the world’s peoples and nations.

This same gap also generates a rationale for development assistance as an instrument to speed-up progress towards achieving the aspirations set forth in the UDHR and for the setting of clear goals, such as the Sustainable Development Goals (SDGs), to monitor progress. Many development institutions, notably the programmes and specialized agencies of the UN, specifically link the rights set forth in the UDHR, their development programmes, and attainment of the SDGs. These links relate both to final goals and to the development processes these institutions seek to put in place to attain development goals (such as the SDGs). The rights-based approach to development is meant to forge these links by specifically targeting ‘the realization of human rights as laid down in the Universal Declaration’ and by applying human rights principles as a guide to ‘all development cooperation and programming in all sectors and in all phases of the programming process’ (UNICEF 2003: 91).

In terms of principles, the UDHR insists that human rights are (among other properties):

1. indivisible—rights have equal status and cannot be ranked in a hierarchical order;
2. inalienable—rights cannot be given up or taken away;
3. interdependent—the realization of one right may depend, at least in part, on the realization of others; and
4. equal—all human beings are entitled to their human rights.

This study is concerned with measurement of progress towards the attainment of human rights, which are inherently multidimensional. It falls within a large literature, both theoretical and empirical, related to the measurement of welfare across multiple dimensions of wellbeing (Alkire et al. 2015; Ravallion 2016). It seeks to make three contributions. First, it assesses whether the main approach currently employed for measuring welfare across multiple dimensions, the multidimensional poverty index (MPI) of Alkire and Foster (2007) (henceforth AF) is, in fact, human rights consistent. We assert that, while elegant and practical, the AF approach is, to a possibly surprising degree, inconsistent with human rights principles.

Second, an alternative approach based on first-order dominance (FOD) principles is presented and assessed (Arndt et al. 2012). The FOD approach is in line with the four human rights principles listed above. However, the gains in terms of consistency with very general human rights principles may imply that consistent welfare ranking becomes indeterminate.

Third, the AF and FOD approaches are applied to data for 26 African countries to determine welfare rankings around the year 2002 and for the most recent data point available (approximately 2012). The AF and FOD approaches provide similar country rankings in both time periods and both point to progress over time in most, though not all, countries. Divergences between the AF and the FOD approaches do occur, and provide significant additional information. The AF and FOD approaches are therefore best viewed as complementary methodologies that should be employed in tandem.

The remainder of this paper is structured as followed. Section 2 presents the AF and FOD approaches and assesses them in the context of a rights-based development approach. Section 3 presents the application to welfare assessment for sub-Saharan Africa (SSA), while Section 4 concludes.

2 AF and FOD

2.1 Review of recent applications

The UNDP MPI (Alkire and Santos 2010) and the underlying AF methodology (Alkire and Foster 2007; Alkire et al. 2015) provide an important avenue for within- and cross-country multidimensional welfare comparisons and for comparisons over time. The MPI is relatively easy to compute from standard DHS (Demographic and Health Surveys) and/or Multiple Indicator Cluster Surveys (MICS) data and is decomposable following the Foster et al. (1984) class of poverty measures. Batana (2013) applies the AF method across 14 SSA countries to study poverty of women in the dimensions of assets, health, schooling, and empowerment. Alkire and Housseini (2014) present an extensive evaluation of multidimensional poverty in 37 SSA countries based on the 2014 MPI and on a modified index capturing severe deprivation. Analysis was extended to subregional decompositions and an assessment of poverty dynamics in 19 countries with consistent time-series.

FOD is also well-suited to within- and cross-country multidimensional welfare comparisons. It is, perhaps, particularly well-suited to welfare comparisons through time. It was applied to Vietnam and Mozambique by Arndt et al. (2012), and FOD has since been applied to numerous countries, including the Democratic Republic of the Congo (DRC), Ghana, Nigeria, Tanzania, and Zambia. The results of the work on the countries just listed are presented in companion volumes examining growth and poverty in SSA (Arndt et al. 2016c) and techniques in poverty measurement (Arndt and Tarp 2016). Also, Permanyer and Hussain (2015) applied FOD analysis to a cross-country study of 38 developing countries and, using the same set of indicators, compared their results to other multidimensional methodologies.

The next two subsections provide a brief review of each approach. More details on the MPI can be found in Alkire et al. (2015). A detailed treatment of the FOD theory and practice can be found in Arndt et al. (2016d) and Arndt and Mahrt (2016), respectively.

2.2 AF

The AF approach aggregates household or individual welfare statuses across multiple dimensions into a single index that provides insight into both the incidence and intensity of poverty. The approach depends upon dual cut-offs that identify dimension-specific deprivation thresholds and an across-dimension threshold (k), which measures the share of weighted indicators in which a household is deprived. Households with weighted deprivation shares greater than k are deemed poor and the proportion of such households yields the multidimensional poverty headcount (H). The method further identifies the intensity of poverty (A) as the average weighted deprivation count among the poor. The final AF poverty index is referred to as the adjusted headcount ratio (M_0), expressed as the product of the headcount ratio and the intensity of poverty:

$$M_0 = HA \quad (1)$$

2.3 FOD

The FOD approach is based on the assumption that it is better to be not deprived than deprived in any welfare dimension. As with AF, FOD requires the identification of welfare indicators and the specification of a threshold value that defines deprived versus not deprived.¹ Hence, AF and FOD measures are both potentially sensitive to indicator thresholds. However, the FOD approach permits comparison of populations without indicator weights. And, in the absence of weights, FOD faces no need to specify poverty line thresholds (the k in the AF approach).

Applying the FOD concept for two multidimensional discrete population distributions, A and B, A dominates B if and only if it is possible to obtain distribution B from distribution A by moving probability mass from better to worse outcomes within A. This methodology builds upon earlier contributions to the dominance literature (Atkinson and Bourguignon 1982; Batana and Duclos 2010; Bourguignon and Chakravarty 2003; Cowell and Victoria-Feser 2007; Duclos and Échevin 2011; Duclos and Makdissi 2005; Duclos et al. 2007, 2011; Gravel and Mukhopadhyay 2010; Gravel et al. 2009; Lehmann 1955; Levhari et al. 1975; Shaked and Shanthikumar 2007). Mosler and Scarsini (1991) and Dyckerhoff and Mosler (1997) present a linear programming method to check FOD in the general multivariate finite case. Arndt et al. (2012) provide an empirical implementation of the linear programming-based method for checking multidimensional FOD with up to seven binary deprivation indicators.

Assume we have n welfare dimensions and for each dimension a 0/1 (deprived/not deprived) valued binary indicator is defined. With five binary indicators ($n = 5$), a total of 32 ($= 2^5$) welfare status combinations are possible for each basic unit of analysis, normally either an individual or a household. Let a_i and b_i be the shares of population A and B, respectively, with welfare status i . Let the variable x_{ij} represent probability mass transfer from welfare status i to welfare status j , where j is an alias of i . Define Z as the set of source-destination pairs ij that move probability from a preferred welfare status i ($i = 1, \dots, 32$) to a less preferred welfare status j .

A welfare status i is preferred to status j if and only if status i is at least as good as status j in all dimensions and better in at least one dimension. For example, status (1,1,1,1,1), indicating not deprived in all dimensions, is preferred to all other welfare statuses. Similarly, status (0,0,0,0,0), indicating deprived in all dimensions, is least preferred. Moving to more complex cases, status (1,1,1,0,0) is preferred to status (0,1,1,0,0) because the latter status can be obtained from the former by removing the not deprived status in dimension one, which makes the unit of analysis

¹ Multiple thresholds for a single indicator are possible but quickly suffer the curse of dimensionality. In practice, both AF and FOD typically rely on binary indicators.

unambiguously worse-off. The criterion is strict. Status (0,1,1,1,1) is not unambiguously better than status (1,0,0,0,0).

Under these conditions, population A dominates population B (i.e. FOD is found) if there exist x_{ij} with:

$$x_{ij} \geq 0, x_{ij} \leq 1, x_{ii} = 0, \text{ such that } a_i + \sum_{j \in Z} x_{ji} - \sum_{j \in Z} x_{ij} = b_i \text{ for all } i \quad (2)$$

While FOD analysis allows for comparison across populations without imposing subjective restrictions, the FOD criterion sometimes leads to indeterminate outcomes, meaning that population A does not dominate population B and population B does not dominate population A. Satisfaction of Equation (2) also gives no sense of the extent of domination. Arndt et al. (2012) suggest mitigating indeterminacy by conducting FOD analysis on repeated bootstrap samples drawn from each population (Efron 1979). Where appropriate, bootstrapping is conducted taking the survey design (strata and clustering) into account. The subsequent FOD results can be interpreted as a measure of likelihood of domination providing substantially more information than FOD in the static case. Furthermore, one can define a measure of net-domination (the average frequency that an area dominates all other populations minus the average frequency that the area is dominated by all other populations), which can be used as a basis to rank populations (Copeland 1951).

2.4 Assessment

The FOD approach clearly conforms better to human rights concepts than the approach propounded by AF. If we view indicators as basic rights such as a right to a primary school education, a right to safe water, a right to adequate sanitation, and so forth, then the weighting scheme integral to AF becomes problematic. Under AF, the indicators (rights) are not *indivisible* in that they are explicitly ranked via the weighting scheme. Under AF, the indicators (rights) are not *inalienable* in that it is perfectly possible to be judged better-off even after a population-wide shift from not deprived to deprived in one indicator if there are gains in other indicators. Under AF, the indicators (rights) are not *equal* in that measured progress is possible despite welfare losses among significant subgroups of the population, which are outweighed by gains among other subgroups. Overall, when focusing on distributions of populations as classified by deprived or not deprived status for multiple binary indicators (rights), the AF approach violates three of the four principles listed in Section 1.

In contrast, the FOD approach violates none. Under FOD, indicators (rights) are *indivisible* in that no weighting of any sort is applied. Under FOD, indicators (rights) are *inalienable* in that, when population A FOD population B, the distribution of indicators (rights) in population A can be obtained from population B uniquely by adding weight from less preferred to more preferred states within population B. Finally, under FOD, indicators are *equal* in that for progress to be deemed to have occurred, it must occur across the full distribution of welfare outcomes. To take just one example, population A will never FOD population B if the share of population A in status (0,0,0,0,0) is greater than the corresponding share in population B. More generally, welfare gains among better-off groups never offset welfare losses among worse-off groups.

These differential properties can lead to dissimilar results by approach for the same indicators and indicator thresholds. A substantial difference is that FOD may result in indeterminate outcomes while AF will always generate a welfare ranking (though these might not be statistically significant). Consider a simple example in three dimensions. Under FOD, dominance cannot be established between a pair of welfare outcomes such as (0,1,0) and (1,0,1). This occurs because, consistent with human rights principles, no assumptions are made about the relative importance of each dimension. With the AF method, a result is always obtained, but the result depends upon how

weights are assigned. With equal weighting across indicators, the second pair is clearly superior to the first, while the weighting scheme (0.2, 0.6, 0.2) leads the first outcome to be associated with greater welfare. Put differently, fidelity to very general human rights principles, as in FOD, may come at a cost in terms of determinacy of outcomes. At the same time, the determinacy of outcomes in AF comes at the cost of violating rights-based principles in the evaluation of welfare.

With respect to the *interdependence* principal, both AF and FOD are effectively agnostic in that neither make any attempt to capture interdependence. From a mathematical perspective, interdependence can be viewed as potentially rendering comparisons less multi-faceted than the number of indicators selected would suggest. The logic is similar to multi-collinearity in econometrics and applies to both approaches. Consider a five-indicator example in which binary indicators 1 and 2 are perfectly correlated. Under FOD, either indicator 1 or indicator 2 could be dropped from the analysis and the results would be the same. Under AF, this is also true as long as the weight accorded to the retained indicator (1 or 2) is the same as the sum of the weights applied to indicators 1 and 2 in the five-indicator case. Reinserting the dropped indicator thus provides no additional information.

There are also practical differences in the application of AF and FOD. These are best discussed in the context of an application.

3 Application to sub-Saharan Africa

Sub-Saharan African economies have seen relatively high growth rates during the last 20 years (Duman and Heise 2010; IMF 2013; Leibfritz and Flaig 2013; Martinez and Mlachila 2013; Page and Arbache 2007). The extent to which growth has improved the living conditions for those living in poverty is less clear; thus, the poverty, growth, and inequality triangle continues to be widely debated (Bourguignon 2004). Here we frame the first aspect of the triangle, poverty, in the context of attaining basic human rights, which is inherently nonmonetary and multidimensional in nature. The expansion of DHS and the United Nations International Children's Emergency Fund's (UNICEF) MICS over the last two decades has greatly enhanced the potential for consistent multidimensional poverty analysis across countries and time. Yet, relatively few studies define a consistent set of welfare measures to assess multidimensional wellbeing across African countries. Sahn and Stifel (2000) apply factor analysis of household socioeconomic characteristics in 15 SSA countries. Booysen et al. (2008) extend this work to seven SSA countries, employing multiple correspondence rather than factor analysis. As mentioned, Batana (2013) applies the AF method in an assessment of women's wellbeing in 14 SSA countries. And, in the most comprehensive analysis, Alkire and Housseini (2014) present an evaluation of 37 SSA countries based on the 2014 MPI as well as a modified index capturing severe deprivation.

We pursue a double goal of first contributing to this literature by identifying welfare rankings of 26 SSA countries and evaluating the extent to which broad-based growth in welfare and changes to welfare rankings have occurred over time. Second, we consider the ability of the FOD and AF methodologies to assess multidimensional poverty from a rights-based perspective. As noted, satisfying the tenets of the UDHR is an ongoing process and this is particularly true in SSA, where access to minimal standards of living such as basic sanitation, clean drinking water, adequate shelter, participation via access to information, and primary education continue to be out of reach for large portions of the population. Being able to assess multidimensional poverty while adhering to basic rights principles (inalienability, indivisibility, interdependence, and equality) is therefore particularly relevant in the SSA context.

3.1 Data

The DHS provide relevant wellbeing data using a harmonized survey structure across most of the 26 SSA countries examined in this study (DHS 2016). Although not all countries in SSA are included, the investigated countries represent 84 per cent of the population. Two DHS surveys were used for each country, representing a year close to 2002 ($t = t_1$) and another year close to 2012 ($t = t_2$), enabling us to monitor welfare changes over time within a given country. The DHS were not available in both years for South Africa and the DRC. As alternatives, the 2002 and 2012 national General Household Surveys (GHS) provides the data for South Africa (Statistics South Africa 2003, 2013) and the 2001 MICS provides the first round of the DRC data (RDdC 2002). The MICS and DHS are highly compatible due to a close collaboration to ensure that survey tools and methodologies are harmonized and comparable.

Sample sizes for each country and year are displayed in Table 1. The median first and last survey years are 2003 and 2012, respectively. The sample size in the second round of surveys is typically higher (median 13,300) than in the first round (median 8,100), but with major variations across countries (approximately between 5,000 and 38,000 observations in each round).

Table 1: Survey year, sample size, and population size

	Survey year			Sample size			Population ^d
	t_1	t_2	Range	t_1	t_2	Total	2012
Benin (BEN)	2001	2011	10	5,756	17,422	23,178	10,049,792
Burkina Faso (BFA)	2003	2010	7	9,075	14,410	23,485	16,590,813
Cameroon (CMR)	2004	2011	7	10,435	14,177	24,612	21,659,488
Chad (TCD)	2004	2014	10	5,358	17,108	22,466	12,715,465
Congo, Dem. Rep. (DRC) ^b	2001	2013	12	8,622	18,144	26,766	70,291,160
Congo, Rep. (COG)	2005	2011	6	5,870	11,610	17,480	4,286,188
Côte d'Ivoire (CIV) ^a	2005	2011	6	4,348	9,649	13,997	21,102,641
Ethiopia (ETH)	2000	2011	11	14,059	16,678	30,737	92,191,211
Gabon (GAB)	2000	2012	12	6,171	9,716	15,887	1,613,489
Ghana (GHA)	2003	2014	11	6,233	11,834	18,067	25,544,565
Guinea (GIN)	2005	2012	7	6,242	7,103	13,345	11,628,767
Kenya (KEN)	2003	2014	11	8,532	36,396	44,928	42,542,978
Lesotho (LSO)	2004	2014	10	8,561	9,402	17,963	2,057,331
Madagascar (MDG)	2004	2008	4	8,412	17,832	26,244	22,293,720
Malawi (MWI)	2000	2010	10	13,648	24,789	38,437	15,700,436
Mali (MLI)	2001	2012	11	12,267	10,104	22,371	16,112,333
Mozambique (MOZ)	2003	2011	8	12,295	13,919	26,214	25,732,928
Namibia (NAM)	2000	2013	13	6,342	9,815	16,157	2,291,645
Nigeria (NGA)	2003	2013	10	7,162	38,361	45,523	168,240,403
Rwanda (RWA)	2000	2014	14	9,650	12,677	22,327	10,817,350
Senegal (SEN)	2005	2012	7	7,359	8,406	15,765	13,780,108
South Africa (ZAF) ^c	2002	2012	10	26,169	24,856	51,025	52,356,381
Tanzania (TZA) ^a	2003	2011	8	6,484	10,019	16,503	48,645,709
Uganda (UGA)	2000	2011	11	7,847	9,028	16,875	35,400,620
Zambia (ZMB)	2001	2013	12	7,110	15,868	22,978	14,786,581
Zimbabwe (ZWE)	2005	2010	5	9,266	9,756	19,022	14,565,482
Median	2003	2012	10	8,130	13,298	22,419	16,351,573

Notes: ^a Côte d'Ivoire 2005, Tanzania 2003, Tanzania 2011 DHS AIS surveys.

^b DRC 2001 MICS.

^c South Africa 2002, 2012 GHS.

^d World Bank (2016b).

Source: own calculations based on DHS standard surveys, except where noted.

3.2 Indicator definitions

Multidimensional analysis in this study is based upon five binary welfare indicators. The choice of indicators draws upon the widely-applied Bristol Indicators (Gordon et al. 2003) and aims to cover basic aspects of wellbeing in a developing country setting—specifically: drinking water, sanitation, shelter, communication, and education. For each indicator, a household is either deemed to be deprived and the indicator is assigned the value 0, or the household is not deprived and the indicator takes the value 1.

Water

A household is not deprived in the water indicator when it obtains drinking water from a safe source (tap, pipe, public standpipe, tube-well, borehole, covered dug well, rainwater, or bottled/sachet water). For monitoring progress in achieving the Millennium Development Goals (MDGs), UNICEF and the World Health Organization (WHO) developed a commonly adopted definition of improved water sources that also classifies covered springs as safe (UNICEF and WHO 2015). However, early rounds of the DHS do not distinguish between protected and unprotected springs. To maintain consistency over time and to avoid confusing progress with households simply shifting between changing survey categories, our definition classifies all springs as a deprivation.

Sanitation

A household is considered not deprived in sanitation when the household has access to sanitation facilities and deprived when the household relies upon buckets, hanging latrines, or open defecation. Our definition differs from that of UNICEF and WHO, which specify deprivation in sanitation to include shared facilities, pit latrines without a slab, and facilities that flush or pour to somewhere other than a sewer system, septic tank, or pit latrine. Four issues in t_1 surveys prevent us from adopting this widely-accepted definition. First, in t_1 improved pit latrines appear to refer to ventilated improved pit latrines in some instances and to any improved pit latrine in others. Second, there is no distinction between traditional pit latrines with and without a slab. Third, the surveys do not specify whether facilities flush to a sewer system, septic tank, or pit latrine. Finally, the 2001 DRC MICS and South African GHS surveys do not specify whether facilities are shared. Consequently, we can only determine with consistency and certainty whether households use any kind of flush toilet or whether households use no facilities. As the latter deprivation more closely corresponds to both policy goals and severe deprivation indicators in other studies, this is the definition adopted here (see Alkire and Housseini 2014; Gordon et al. 2003).

Shelter

A household is not deprived in shelter if the floor of its dwelling is constructed from solid material, and is deprived when the floor is derived from non-permanent material. Solid floors primarily include cement and brick, and to a lesser degree tile, carpet, vinyl, wood, iron, and parquet. Inadequate flooring mainly includes earth, palm, and other similar types of flooring materials. The South Africa GHS does not report flooring material and consequently we define shelter deprivation in terms of wall quality.

Communication

The inclusion of access to information and communication in the SDGs highlights their critical role in ‘sustained, inclusive and equitable economic growth and sustainable development, competitiveness, access to information and knowledge, poverty eradication and social inclusion’ (UNGA 2015: 4). Our communication indicator measures radio, television, and telephone (cellular

or fixed) ownership.² If the household does not possess any of these communication methods, it is deemed deprived.³

Education

As mentioned, the UDHR identifies education as a basic human right. It is also an important indicator of the human capital level within a household. A household is not deprived in education if one or more household members have completed at least six years of schooling. Among the countries in this study, six years of schooling is the median number of school years required to complete primary school.

Indicator sensitivity

To evaluate the sensitivity of FOD outcomes to the water and sanitation indicator definitions, we specify alternative thresholds in t_2 that correspond to the UNICEF and WHO definitions of improved water and sanitation described above. The improved sanitation indicator is defined with and without considering whether the facility is shared. Sensitivity to indicator thresholds in FOD analysis is also explored by Ajakaiye et al. (2016), Arndt et al. (2016b), and Mahrt and Masumbu (2016) in applications to Nigeria, Tanzania, and Zambia respectively. The authors find that FOD temporal dynamics can be sensitive to indicator thresholds, while spatial patterns are typically more robust.

3.3 Results

Welfare indicator averages and the joint distribution of welfare

Overall, we see progress in most countries for most indicators, but backsliding within welfare dimensions is also observed (Table 2). Three countries experience welfare reductions in two dimensions: Côte d'Ivoire and Nigeria (sanitation and shelter) and Rwanda (water and sanitation). Seven countries experience backsliding in one indicator: Cameroon, the Republic of the Congo, Gabon, and Tanzania (sanitation); DRC and Madagascar (shelter); and Burkina Faso (education). The remaining 16 countries see progress in all five dimensions. The degree of progress varies substantially by dimension and area. For example, Ethiopia, which starts from a very low base, achieves impressive progress in every dimension, with the percentage of the population not deprived in water, sanitation, and communication more than doubling. In contrast, South Africa, which starts from a high base, achieves relatively modest gains in every indicator. Considering the 26 SSA countries in aggregate, there is no backsliding (of weighted averages) in any dimension. Population-weighted averages improve for the aggregate in access to safe water (47 to 61 per cent),

² While the SDGs emphasize the importance of internet access, information on internet access is only collected in select country surveys in later years. However, a sufficiently high correlation exists between mobile phone ownership and internet access that we do not feel this presents a deficiency in the communication indicator. For instance, in countries for which the data are available, the share of households with internet access that did not also have a mobile phone ranged from 0 per cent in Côte d'Ivoire and Burkina Faso to 6 per cent in Benin, with most countries at 1 per cent or less.

³ The 2000 Malawi DHS does not report phone possession. An examination of the 2004 Malawi DHS suggests that phone ownership is highly related to radio ownership. In a sample of 13,664, of households that do not own a radio, only 18 possess a phone. The 2001 DRC MICS does not report phone or television ownership in rural areas. The 2007 DRC DHS, with a sample of 5,189 households, indicates that 3,482 rural households do not own a radio but only three of these households own a television and only 27 own a mobile or fixed phone. Therefore, we do not consider this lack of reporting to be a significant limitation.

sanitation (68 to 75 per cent), shelter (43 to 45 per cent), communication access (62 to 79 per cent), and education (62 to 70 per cent).

Table 2: Individuals not deprived in welfare indicators, in t_1 and t_2 (per cent)

	Water		Sanitation		Shelter		Communication		Education	
	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2
Benin	66	75	32	41	59	61	77	86	39	56
Burkina Faso	58	76	30	36	40	44	68	82	34	32
Cameroon	59	63	93	92	49	54	68	81	70	74
Chad	36	56	28	30	5	9	41	71	28	42
Congo, D.R.	28	34	81	87	21	20	34	61	66	83
Congo, Rep.	58	73	90	90	58	67	64	90	90	90
Côte d'Ivoire	69	78	69	66	82	82	74	90	57	59
Ethiopia	19	42	18	62	7	11	22	49	23	44
Gabon	79	89	98	91	78	88	82	97	85	90
Ghana	66	88	74	79	86	93	75	93	74	83
Guinea	62	75	72	83	46	55	69	84	37	50
Kenya	42	57	81	87	34	45	76	91	84	90
Lesotho	73	77	56	71	59	65	58	91	85	90
Madagascar	32	37	54	56	40	37	61	65	39	41
Malawi	66	79	84	90	20	24	59	66	58	68
Mali	44	66	79	88	21	27	75	90	23	40
Mozambique	46	53	53	61	26	27	61	67	35	49
Namibia	77	84	41	49	42	56	76	95	86	91
Nigeria	40	63	75	70	65	61	76	89	72	73
Rwanda	42	40	97	97	15	25	40	77	50	63
Senegal	69	78	78	82	65	73	91	98	43	62
South Africa	86	93	86	95	81	89	88	99	96	97
Tanzania	51	54	89	86	27	32	61	80	85	88
Uganda	52	70	85	90	20	27	56	81	66	72
Zambia	53	63	73	84	39	44	50	81	81	86
Zimbabwe	75	76	68	71	66	69	53	73	94	96

Source: see Table 1.

We now focus on the joint welfare distribution at the household level using two countries as examples. The complete joint distribution of welfare outcomes for Uganda and Nigeria are presented in Table 3. Comparing the distributions for period t_1 , Nigeria has a significantly higher rate of those not deprived in any of the five indicators (24.7 versus 12.2 per cent), but only a slightly lower share deprived in all indicators (3.0 versus 3.2 per cent). The large majority of the distribution involves intermediate outcomes.

Table 3: Sample joint distribution of welfare indicators for Nigeria and Uganda, t_1 and t_2

Outcome ranking	Welfare indicator combinations ^a					Distribution (per cent)					
	1	2	3	4	5	Nigeria			Uganda		
						t_1	t_2	Change	t_1	t_2	Change
Worst outcome	0	0	0	0	0	3.0	2.2	-0.7	3.2	1.1	-2.1
	0	0	0	0	1	2.4	1.0	-1.4	2.2	0.3	-1.9
	0	0	0	1	0	3.0	3.4	0.4	1.0	0.5	-0.5
	0	0	0	1	1	3.4	4.2	0.8	1.3	0.7	-0.6
	0	0	1	0	0	0.6	0.3	-0.3	0.1	0.0	-0.1
	0	0	1	0	1	1.4	0.4	-1.0	0.0	0.0	0.0
	0	0	1	1	0	0.6	0.9	0.3	0.0	0.0	0.0
	0	0	1	1	1	4.1	4.1	0.0	0.0	0.1	0.1
	0	1	0	0	0	4.0	1.5	-2.6	9.0	2.4	-6.5
	0	1	0	0	1	2.1	0.6	-1.5	7.7	2.3	-5.5
	0	1	0	1	0	4.6	4.4	-0.2	5.5	5.1	-0.3
	0	1	0	1	1	5.1	4.3	-0.8	13.2	12.2	-1.0
	0	1	1	0	0	1.3	0.4	-0.8	0.1	0.1	0.0
	0	1	1	0	1	2.5	0.2	-2.2	0.7	0.2	-0.5
	0	1	1	1	0	3.9	2.4	-1.5	0.3	0.5	0.2
	0	1	1	1	1	17.7	6.4	-11.2	3.7	4.4	0.7
	1	0	0	0	0	0.9	1.0	0.2	3.4	2.7	-0.7
	1	0	0	0	1	0.6	0.5	-0.2	2.2	1.2	-1.0
	1	0	0	1	0	0.5	1.7	1.2	0.6	1.2	0.5
	1	0	0	1	1	1.0	2.6	1.6	1.1	1.8	0.6
	1	0	1	0	0	0.2	0.1	0.0	0.0	0.0	0.0
	1	0	1	0	1	0.7	0.4	-0.3	0.1	0.0	-0.1
	1	0	1	1	0	0.2	0.7	0.5	0.0	0.0	0.0
	1	0	1	1	1	2.0	6.7	4.7	0.1	0.2	0.1
	1	1	0	0	0	1.4	1.2	-0.2	5.6	4.0	-1.6
	1	1	0	0	1	0.5	0.6	0.1	7.4	4.2	-3.2
	1	1	0	1	0	1.1	3.4	2.3	4.2	8.2	4.0
	1	1	0	1	1	1.8	6.3	4.5	12.5	25.1	12.6
	1	1	1	0	0	0.7	0.4	-0.3	0.3	0.2	-0.1
	1	1	1	0	1	1.5	0.5	-1.0	1.7	0.6	-1.1
	1	1	1	1	0	2.5	3.2	0.7	0.6	1.4	0.8
Best outcome	1	1	1	1	1	24.7	33.8	9.1	12.2	19.3	7.0
Sum						100	100	0	100	100	0
Avg. number of good outcomes	Equal weights					3.3	3.6	0.3	2.8	3.4	0.6
	Unequal weights ^b					3.0	3.4	0.4	3.2	3.8	0.6

Notes: ^a Column headings 1, 2, 3, 4, 5, refer to water, sanitation, shelter, communication, and education, respectively.

^b Weighting scheme of (2, 2, 1/3, 1/3, 1/3).

Source: see Table 1.

Without imposing weights, the relative importance of dimensions cannot be ranked. Thus, the relative welfare of a household deprived only in sanitation compared to a household deprived only in water or even compared to a household deprived in all dimensions except sanitation, is

indeterminate. If we simply count the number of indicators for which a household is not deprived, giving each indicator equal weight, the average number of positive welfare outcomes in Nigeria is 3.3 and 3.6, compared to 2.8 and 3.4 in Uganda, in t_1 and t_2 respectively. However, imposing alternative weighting schemes may reverse estimated relative welfare. For example, placing higher priority on water and sanitation outcomes results in Nigeria attaining fewer positive outcomes (3.0, 3.4) than Uganda (3.2 and 3.8) in t_1 and t_2 . While imposing weights facilitates comparability of outcomes, it violates the principle of indivisibility (all welfare dimensions have equal status and cannot be ranked in a hierarchical order). Furthermore, outcomes from weighted analysis have the undesirable property of depending on the choice of weights. The FOD approach applied in this study eliminates the need for weighting schemes producing conclusions that are both robust to the choice of weights and consistent with the rights-based approach.

Temporal FOD

Temporal FOD analysis provides evidence of multidimensional welfare performance over time. FOD tests lead to three possible outcomes for each country: t_2 dominates t_1 , t_1 dominates t_2 , or no dominance. Static outcomes derived from the actual survey data provide little information in the case of indeterminacy and no information on the extent of domination. Determining the frequency of domination across 100 bootstrap samples helps fill these gaps.

Table 4 presents static and bootstrap FOD results. Sixteen countries experienced broad-based progress in welfare over time and exhibited a high degree of correlation between static and bootstrap outcomes. The average frequency (normalized to the interval [0,1] henceforth) of domination in the bootstrap is 0.92 when static FOD occurs. Eight countries achieved FOD in all 100 bootstraps (Ethiopia, Guinea, Kenya, Mali, Namibia, South Africa, Uganda, and Zambia) and six achieved FOD in more than 80 bootstraps (Ghana, Malawi, Senegal, Benin, Lesotho, and Chad). Despite static FOD, bootstrap sampling provides weaker likelihood of advancement in Mozambique (0.57) and Zimbabwe (0.48), though both periods of analysis cover a shorter timespan of eight and five years, respectively.

Table 4: Degree of temporal static FOD and temporal bootstrap FOD

	$t_2 - t_1$	Static FOD	Bootstrap FOD normalized frequency	Average bootstrap normalized frequency
Ethiopia	11	1	1	0.92
Guinea	7	1	1	
Kenya	11	1	1	
Mali	11	1	1	
Namibia	13	1	1	
South Africa	10	1	1	
Uganda	11	1	1	
Zambia	12	1	1	
Ghana	11	1	0.99	
Malawi	10	1	0.99	
Senegal	7	1	0.99	
Benin	10	1	0.93	
Lesotho	10	1	0.91	
Chad	10	1	0.82	
Mozambique	8	1	0.57	
Zimbabwe	5	1	0.48	
Cameroon	7		0.20	0.08
Congo, D.R.	12		0.17	
Burkina Faso	7		0.14	
Madagascar	4		0.08	
Tanzania	8		0.06	
Congo, Rep.	6		0.04	
Côte d'Ivoire	6		0.03	
Rwanda	14		0.03	
Gabon	12		0	
Nigeria	10		0	

Source: see Table 1.

The average bootstrap frequency of domination for the 10 countries with indeterminate static FOD is only 0.08. Though a few countries have weak evidence of advancement, Cameroon (0.20), the DRC (0.17) and Burkina Faso (0.14), in most the frequencies are very small (Madagascar, Tanzania, the Republic of Congo, Côte d'Ivoire, and Rwanda) or zero (Gabon and Nigeria). Consistent with generally positive trends becoming more easily identifiable over longer periods of time, the median timespan for countries with static FOD is 10 years compared to 7.5 for countries without static FOD. In no case (static or bootstrap) did the first period dominate the second, indicating no broad-based decline in welfare over time for any of the 26 analysed SSA nations.

In measuring broad-based progress, temporal FOD adheres to basic rights principles in that if any segment of the population (equality) is regressing in any dimension (inalienability), FOD will not register advancement. All cases of temporal stagnation are accompanied by backsliding in at least one welfare indicator (see Table 2). Of particular note are Gabon and Cameroon, in which stagnation was driven by diminished access to sanitation facilities. Despite this reduction, both countries remain at the top in terms of second-period sanitation welfare. Furthermore, both countries achieved significant progress in all other indicators. These results draw attention to the need to interpret FOD outcomes with a great degree of discernment. FOD stagnation does not necessarily imply a lack of any progress. Instead, it indicates that the progress was not achieved throughout the distribution and among all indicators. Overall, we find ample evidence of robust multidimensional progress in SSA and no evidence of multidimensional regress over the period analysed.

Spatial FOD

As with temporal comparisons, spatial FOD tests are conducted using both the static and bootstrapping approaches. Spatial FOD outcomes are the result of country-by-country comparisons, whereby country A dominates country B, country B dominates country A, or no country dominates. Tables 5 and 6 present bootstrap frequencies that a given row country first-order dominates the corresponding column country. Static FOD exists except when the bootstrap entry is underlined. Generally static FOD is observed where there are higher probabilities of bootstrap FOD (between 0.37 and 1) compared to no static FOD (between 0 and 0.45). Row averages represent the frequency that the row country dominates all other countries while column averages indicate the frequency the column country is dominated by all other countries.

Almost all domination in both periods occurs when Gabon or South Africa dominate or when Burkina Faso, Chad, Ethiopia, Madagascar, and Mozambique are dominated. The remaining countries experience a greater occurrence of indeterminate outcomes. Indeterminate outcomes occur when two countries are either very similar or very different in the distribution of welfare in each indicator. Consistent with the principle of indivisibility and as noted in the discussion of Table 3, when welfare outcomes follow distinctly different patterns, FOD often cannot be determined. For example, in the second period Rwanda exhibits a particularly high degree of indeterminacy—it only dominates Ethiopia in bootstrap samples and is never dominated. This outcome likely stems from a unique pattern of relatively low water and shelter welfare coupled with the highest welfare in sanitation.

Country rankings

Table 7 presents country rankings in t_1 and t_2 based on the FOD and AF methodologies. In the static case, spatial FOD tests allow two countries to be ranked only when static results are determinate; however, bootstrap frequencies of net-domination enable a complete ranking of all countries (for a sufficient number of bootstrap repetitions). FOD net-domination measures the average frequency across all bootstrap samples that a country dominates all other countries minus the average frequency that it is dominated by all other countries (Tables 5 and 6 average bootstrap column values minus average bootstrap row values). To facilitate comparisons with the AF adjusted headcount ratio, M_0 , we create the FOD domination score by linearly transforming FOD net-domination. Both the FOD domination score and M_0 have a range of [0,1], where higher levels of multidimensional welfare correspond to lower values.⁴ Following Alkire and Housseini's (2014)

⁴ Shading draws attention to clusters in which sequential countries t_2 do not out-perform one another by more than two FOD domination points. It is therefore difficult to distinguish between differences in welfare and variability introduced through bootstrapping.

MPI analysis of SSA, the AF poverty threshold (k) identifies those deprived in at least one-third of weighted indicators as multidimensionally poor. In this study, indicators are assigned equal weight, and as such, $k = 2$ deprivations.

Table 5: Degree of bootstrap spatial FOD, $t = t_1$

	ETH	TCD	MDG	MOZ	BFA	DRC	UGA	GIN	ZMB	BEN	MLI	NGA	MWI	KEN	LSO	TZA	NAM	ZWE	CMR	RWA	CIV	GHA	SEN	COG	GAB	ZAF	AVG		
ETH																											0		
TCD																												0	
MLI	0.39																											0.02	
DRC	1																											0.04	
MDG	1	<u>0.12</u>																										0.04	
RWA	1	<u>0.35</u>																										0.05	
BFA	1	0.90																										0.08	
MOZ	1	0.93																										0.08	
ZMB	1	1																										0.08	
MWI	1	1																										0.08	
ZWE	1	1																										0.08	
LSO	1	1	<u>0.05</u>																									0.08	
UGA	1	1				<u>0.10</u>																						0.08	
KEN	1	0.96		<u>0.02</u>		<u>0.35</u>																						0.09	
BEN	1	0.97			0.71																							0.11	
NAM	1	1			0.79																							0.11	
NGA	1	0.93	0.99			<u>0.01</u>																						0.12	
GIN	1	1	<u>0.06</u>	0.75	0.37																							0.13	
TZA	1	1		<u>0.04</u>		0.98	<u>0.21</u>																					0.13	
CIV	1	1	1	1	0.95			<u>0.09</u>		<u>0.01</u>																		0.20	
GHA	1	1	1	1	1			<u>0.27</u>		<u>0.01</u>																		0.21	
CMR	1	1	0.99	1	<u>0.11</u>	0.99	0.94																					0.24	
SEN	1	1	0.99	1	1			0.98		0.61	<u>0.28</u>																	0.27	
COG	1	1	0.91	0.98		1	0.48		0.97							0.66												0.28	
GAB	1	1	1	1	1	1	1	1	1	0.77	1	1	1	0.67	<u>0.24</u>	0.51	<u>0.01</u>		1	0.85	<u>0.01</u>							0.64	
ZAF	1	1	1	1	1	1	0.79	1	1	1	1	1	0.99	1	1	<u>0.04</u>	1	1			<u>0.21</u>							0.68	
AVG	0.94	0.81	0.32	0.31	0.28	0.22	0.14	0.13	0.12	0.10	0.09	0.08	0.08	0.07	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.01	0	0	0	0	0	0	0.15

Notes: Row country dominates column country. Static FOD static domination occurs with bootstrap domination except when underlined. Countries are sorted by the AVG column (ascending) and row (descending). Country abbreviations are listed in Table 1.

Source: see Table 1.

Table 6: Degree of bootstrap spatial FOD, $t = t_2$

	ETH	TCDD	MOZ	MDG	BFA	BEN	DRC	NGA	ZMB	CIV	TZA	GIN	MLI	MWI	UGA	KEN	LSO	COG	CMR	SEN	NAM	ZWE	GAB	RWA	GHA	ZAF	AVG	
ETH																												0
TCDD																												0
MDG																												0
BFA																												0
DRC	<u>0.01</u>																											0
MWI	<u>0.04</u>	<u>0.04</u>																										0
MOZ	<u>0.14</u>																											0.01
RWA	<u>0.38</u>																											0.02
MWI	1																											0.04
BEN		1			<u>0.24</u>																							0.05
TZA	1	<u>0.12</u>	0.59	<u>0.01</u>			<u>0.21</u>																					0.08
NAM		1			1																							0.08
UGA	1	1	<u>0.16</u>																									0.09
GIN	1	1	0.58	1	<u>0.08</u>																							0.15
KEN	1	0.83	0.97	1							<u>0.06</u>																	0.15
ZWE	1	0.96	1	1																								0.16
NGA	1	1	1	1																								0.16
ZMB	1	1	1	1																								0.16
CMR	1	1	1	1																								0.16
LSO	1	1	1	1	0.64	0.74		<u>0.06</u>																				0.22
CIV	0.98	1	0.97	1	0.77	0.91																						0.23
SEN	1	1	1	1	0.71	0.94						<u>0.33</u>																0.24
COG	1	1	1	1	<u>0.01</u>		0.99	<u>0.04</u>	1		0.81	<u>0.05</u>				<u>0.01</u>												0.28
GHA	1	1	1	1	1	1		0.97		1																		0.32
GAB	1	1	1	1	1	1	1	1	1	0.91	0.97	1	0.99	0.96	0.81	<u>0.45</u>	<u>0.36</u>	<u>0.26</u>	<u>0.11</u>	<u>0.01</u>								0.63
ZAF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	1	1	1	1	0.67				0.91
AVG	0.66	0.64	0.53	0.52	0.26	0.22	0.13	0.12	0.12	0.12	0.11	0.10	0.08	0.08	0.07	0.06	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0	0	0		0.16

Notes: see Table 5.

Source: see Table 1.

Table 7: Country ranks by FOD domination scores and M_0

	$t = t_1$				$t = t_2$					
	FOD dom. score	Rank	M_0	Rank	FOD dom. score	Rank	Change	M_0	Rank	Change
South Africa**	0.16	1	0.09	1	0.05	1	0	0.03	1	0
Gabon**	0.18	2	0.12	2	0.20	2	0	0.05	2	0
Ghana*	0.39	5	0.20	3	0.34	3	-2	0.09	3	0
Congo, Rep.*	0.36	3	0.23	4	0.39	4	1	0.14	4	0
Senegal	0.36	4	0.26	7	0.40	5	1	0.16	5	-2
Lesotho*	0.48	12	0.30	10	0.42	6	-6	0.17	6	-4
Zimbabwe	0.48	10	0.25	5	0.44	7	-3	0.19	7	2
Cameroon*	0.40	6	0.28	8	0.44	8	2	0.23	11	3
Côte d'Ivoire*	0.40	7	0.25	6	0.45	9	2	0.20	8	2
Kenya*	0.49	13	0.32	11	0.45	10	-3	0.21	9	-2
Guinea	0.50	17	0.39	16	0.47	11	-6	0.26	15	-1
Namibia**	0.46	9	0.33	12	0.48	12	3	0.21	10	-2
Zambia*	0.52	18	0.38	14	0.48	13	-5	0.25	13	-1
Nigeria*	0.48	11	0.29	9	0.48	14	3	0.24	12	3
Rwanda	0.49	14	0.49	19	0.49	15	1	0.36	20	1
Uganda	0.53	19	0.40	17	0.49	16	-3	0.26	14	-3
Tanzania	0.46	8	0.33	13	0.52	17	9	0.27	16	3
Malawi	0.50	16	0.39	15	0.52	18	2	0.29	17	2
Mali	0.54	20	0.49	20	0.54	19	-1	0.33	19	-1
Congo, D.R.	0.59	21	0.52	22	0.56	20	-1	0.40	21	-1
Benin	0.49	15	0.42	18	0.59	21	6	0.32	18	0
Burkina Faso	0.60	22	0.52	21	0.63	22	0	0.43	22	1
Madagascar	0.64	24	0.53	23	0.76	23	-1	0.50	24	1
Mozambique	0.62	23	0.54	24	0.76	24	1	0.46	23	-1
Chad	0.90	25	0.71	25	0.82	25	0	0.56	25	0
Ethiopia	0.97	26	0.82	26	0.83	26	0	0.57	26	0
Average	0.50	13.5	0.38	13.5	0.50	13.5	0	0.28	13.5	0
FOD- M_0 correlation	0.95				0.95					
Spearman rank correlation		0.95				0.97				

Notes: Sorted by t_2 FOD domination scores. Shaded areas highlight t_2 consecutive rankings based on very small differences in FOD domination scores. Care must be taken in interpretation due to variability introduced through bootstrapping.

Countries are low-income except: * lower middle-income country; ** upper middle-income country.

Source: see Table 1.

Both methods identify the same sets of countries with the highest (South Africa, Gabon, Ghana, the Republic of Congo, and Senegal) and lowest (Mozambique, Madagascar, Chad, Ethiopia, Burkina Faso, and the DRC) multidimensional welfare. FOD and AF also indicate large clusters of countries with similar outcomes. In t_2 half of all countries fall within a 10-point FOD domination score range [0.44 and 0.54] and a 14-point M_0 range [0.19, 0.33]. Despite the vastly different methodologies, the FOD domination score and M_0 have a correlation coefficient of 0.95 in both periods, and a Spearman rank correlation coefficient of 0.95 and 0.97 in t_1 and t_2 , respectively. This high degree of correlation is consistent with applications by Arndt et al. (2016a) to Mozambican census data, Arndt et al. (2016b) to Tanzanian DHS data, and Permanyer and Hussain (2015) to DHS data in 38 countries.

Changes in welfare rankings over time must be interpreted carefully due to the large clusters of countries with very similar FOD domination scores and M_0 . A notable degree of rank persistence occurs, particularly with the AF methodology. Welfare rankings changed substantially between t_1 and t_2 in only a few countries. With the AF approach, only Lesotho moves more than a few ranks. Consistent with strong evidence of FOD temporal advancement, FOD domination scores indicate Lesotho, Guinea, and Zambia improve six, six, and five places respectively. Tanzania, which has no evidence of FOD temporal advancement, declined nine places. Despite high likelihood of FOD temporal advancement, Benin fell from a rank of 15 to 21. Though Benin advanced in every indicator, its relative welfare in indicators including communication advanced more slowly than in Lesotho, Côte d'Ivoire, and the Republic of Congo, which resulted in more cases of Benin being dominated. This example and the somewhat greater degree of rank fluctuation with the FOD methodology in general likely stems from FOD's strict consideration of the full welfare distribution. Comparisons of FOD and AF outcomes are discussed in more detail in the next section.

FOD versus AF

This section highlights how FOD's strict adherence to the principles of inalienability and equality results in divergent FOD and AF outcomes. South Africa ranks highest in both time periods using both methodologies. However, FOD indicates a widening disparity between South Africa and nearly all other countries, compared to a convergence that occurs in AF. This difference highlights a fundamental distinction between the AF and FOD approaches. FOD requires domination in all dimensions, which occurred less frequently for South Africa in t_1 . For example, South Africa outperformed Gabon in all dimensions except sanitation in t_1 to such a degree that all bootstrap comparisons between South Africa and Gabon are indeterminate (Table 5). Relative welfare gains in South African sanitation result in South Africa dominating Gabon in 67 bootstrap comparisons in t_2 (Table 6). Similar scenarios occur between South Africa and other top-performing countries, including the Republic of Congo, Senegal, and Cameroon. Consequently, the FOD domination score captures a widening domination by South Africa in terms of every welfare dimension, whereas AF indicates an overall catching up by other countries in terms of average incidence and intensity of deprivations.

A similar difference between FOD and AF exists when comparing temporal outcomes. In contrast to temporal FOD (Table 4), the AF method indicates progress in every country in terms of reduced M_0 . Greater reductions in M_0 generally correspond to higher frequencies of FOD improvement. Notable differences include Rwanda and the DRC, which reduce M_0 by 0.13 and 0.14 without FOD advancement, while South Africa improves M_0 by only 0.05 but achieves FOD advancement in every bootstrap iteration. The finding of stagnation in 10 countries using FOD and progress in all countries using AF stems from M_0 's derivation from average poverty incidence and intensity levels compared to FOD's strict criteria that advancement must occur in all indicators and throughout the distribution. The methods are complementary in that M_0 provides a sense of overall trends while FOD, in a sense, establishes a lower bound consistent with the basic human rights framework. Temporal FOD is difficult to achieve, but when it occurs it robustly indicates progress without neglecting any welfare dimensions or any segment of the population. While FOD is more consistent with the principles of basic human rights, both measures provide useful information regarding relative welfare and welfare advancement.

Middle-income countries

The asterisks in Table 7 indicate upper (double asterisk) and lower middle-income (single asterisk) country status.⁵ A strong correspondence exists between middle-income status and FOD and M_0 rankings in both time periods. Senegal and Zimbabwe are exceptions; they are not middle-income countries but outperform some middle-income countries in both time periods. In t_1 , Zambia had not yet achieved middle-income status and had a relatively poor FOD rank of 18 among the 26 countries studied here. Though it exhibited a strong likelihood of rank advancement between periods, Zambia remained at the bottom of the middle-income countries in t_2 , which reflects persistent distributional challenges (African Development Bank 2010). Nigeria (Africa's most populous nation) does not exhibit progress in the temporal analysis. Moreover, it ranks poorly among middle-income countries in t_2 . This weak performance can be attributed to relatively low welfare in water and sanitation, backsliding in sanitation and flooring, and stagnation in education. Nigeria has not been able to approach the other major oil exporters, Cameroon and Gabon (see also Collier et al. (2008) on Nigeria's performance and challenges).

Correlations with other welfare measures

Table 8 presents several common measures of wellbeing and their associated rankings for the 26 study countries. These measures include the World Bank's US\$1.90 and US\$3 per day poverty headcount ratios, gross domestic product (GDP), average annual GDP per capita growth, UNDP's Human Development Index (HDI), and UNDP's MPI, which measures M_0 for an internationally standardized set of indicators. Excluding GDP growth, the rankings suggest that for a given country this collection of indicators might tell very similar (Gabon, Ghana, Mozambique, and South Africa) or very different stories (Chad, Ethiopia, and Madagascar).

⁵ Lower middle-income economies are those with a gross national income (GNI) per capita between US\$1,026 and US\$4,035, and upper middle-income economies are those with a GNI per capita between US\$4,036 and US\$12,475 (World Bank 2016a).

Table 8: Common welfare measures (2012 except where noted)

	US\$1.90 poverty rate ^a		US \$3.20 poverty rate ^a		GDP per capita		GDP per capita growth ^b		HDI		MPI ^a	
	(%)	Rank	(%)	Rank	Constant 2010 US\$	Rank	(%)	(%)	Index	Rank	Index	Rank
Benin	53.1	18	75.6	16	747	17	0.6	21	0.48	16	0.31	14
Burkina Faso	43.7	15	74.7	15	614	20	3.1	13	0.39	25	0.54	24
Cameroon	24.0	5	43.5	3	1,188	9	0.8	20	0.50	11	0.25	9
Chad	38.4	14	64.8	10	913	13	6.0	2	0.39	26	0.55	25
Congo, D.R.	77.1	25	90.7	26	334	26	2.4	15	0.42	21	0.40	21
Congo, Rep.	37.0	12	59.6	9	3,007	4	1.8	18	0.58	5	0.18	6
Côte d'Ivoire	29.0	7	55.1	7	1,248	8	-0.5	24	0.45	18	0.31	16
Ethiopia	33.5	8	71.3	14	392	25	6.1	1	0.43	20	0.56	26
Gabon	8.0	1	24.43	1	10,031	1	0.0	23	0.67	1	0.07	2
Ghana	25.2	6	49.04	6	1,570	6	4.3	8	0.57	6	0.16	5
Guinea	35.3	11	68.7	13	440	23	0.4	22	0.41	23	0.46	23
Kenya	33.6	9	58.9	8	1,043	11	1.9	17	0.54	7	0.19	7
Lesotho	59.7	20	77.3	19	1,162	10	3.2	12	0.48	13	0.16	4
Madagascar	77.8	26	90.5	25	409	24	-0.5	25	0.51	9	0.36	18
Malawi	70.9	24	87.6	24	474	21	2.5	14	0.43	19	0.27	11
Mali	49.3	17	77.7	20	794	15	5.4	4	0.41	22	0.46	22
Mozambique	68.7	23	87.5	23	453	22	4.5	6	0.41	24	0.39	20
Namibia	22.6	4	45.7	5	5,436	3	3.5	10	0.62	3	0.19	8
Nigeria	53.5	19	76.5	18	2,399	5	6.0	3	0.51	10	0.30	13
Rwanda	60.4	21	80.7	22	618	19	5.4	5	0.48	14	0.26	10
Senegal	38.0	13	66.3	12	997	12	1.0	19	0.46	17	0.31	15
South Africa	16.6	2	34.7	2	7,564	2	2.0	16	0.66	2	0.04	1
Tanzania	46.6	16	76.1	17	754	16	3.5	11	0.51	8	0.33	17
Uganda	34.6	10	65.0	11	653	18	4.0	9	0.48	15	0.37	19
Zambia	64.4	22	78.9	21	1,557	7	4.5	7	0.58	4	0.28	12
Zimbabwe	21.4	3	45.5	4	800	14	-3.4	26	0.49	12	0.13	3

Notes

^a Most recent survey.

^b Average annual growth over the period 2002–12.

Source: UNDP (2016), World Bank (2016b), and own calculations.

Table 9 presents correlation and Spearman rank correlation coefficients between the FOD domination score and each welfare measure. The FOD domination score is most highly correlated with the other measure of multidimensional welfare, the MPI (0.82), and the HDI (-0.72), and to a lesser degree with GDP per capita (-0.67) and the US\$3.20 per day poverty headcount ratio (0.66). FOD is only moderately correlated with the often-cited US\$1.90 poverty headcount ratio (0.53), which reiterates the need for triangulation in poverty analysis.

Table 9: Correlations between FOD and common welfare measures in survey closest to 2012 (Spearman rank correlations in parentheses)

	FOD	US\$1.90	US\$3.20	GDP per capita	HDI	MPI
FOD	1					
US\$1.90	0.53 (0.59)	1				
US\$3.20	0.66 (0.65)	0.94 (0.97)	1			
GDP per capita	-0.67 (-0.75)	-0.58 (-0.56)	-0.73 (-0.67)	1		
HDI	-0.72 (-0.65)	-0.48 (-0.39)	-0.66 (-0.44)	0.81 (0.72)	1	
MPI	0.82 (0.82)	0.37 (0.41)	-0.59 (-0.49)	-0.61 (-0.71)	-0.82 (-0.80)	1

Source: see Table 1 and Table 7.

These results are consistent with findings of two recent cross-country studies of multidimensional poverty in SSA. Batana (2013) finds low and positive Spearman rank correlations between M_0 and both monetary and asset poverty headcounts, and negative and moderate to high correlations between M_0 and HDI. Alkire and Housseini (2014) consider the relationship between multidimensional poverty and monetary poverty dynamics. Their analysis finds that, among the countries with both two periods of consistent MPI ratios and US\$1.25 per day poverty ratios, half the countries reduced multidimensional poverty faster than income poverty. They also found a low correlation between annualized reductions in M_0 and annualized growth in GNI.

Indicator sensitivity

Greater detail in recent DHS questionnaires allows us to specify improved water and sanitation indicators in t_2 following UNICEF and WHO (2015) thresholds and to evaluate indicator sensitivity.⁶ The default water indicator classifies tap, pipe, public standpipe, tube-well, borehole, covered dug well, rainwater, or bottled/sachet water as non-deprived sources. The improved water indicator also includes protected springs as non-deprived. The default sanitation indicator classifies households using any sanitation facility as non-deprived. The improved sanitation facilities indicator excludes pit latrines without a slab and facilities that flush or pour to somewhere other than a sewer system, septic tank, or pit latrine. We define improved sanitation with and without the criterion that improved sanitation is not shared.

Table 10 presents average welfare levels based on original water and sanitation indicators and improved water and sanitation indicators (with and without the shared sanitation criterion). In the majority of countries, using the improved water indicator results in virtually no change in terms of the share of population classified as not deprived in this indicator. However, in Cameroon, the DRC, Ethiopia, Kenya, and Rwanda, increased welfare is significant at 6, 17, 9, 10, and 32 percentage points, respectively. Thus, the population-weighted aggregate welfare increases from 58.4 per cent using the original water indicator to 62.9 per cent with the improved water indicator. In contrast, welfare in sanitation is profoundly different both in terms of levels and relative welfare between the countries. Both modifications to the sanitation indicator significantly decrease aggregate sanitation welfare from 73.9 per cent to 40.7 per cent and 23.9 per cent using the improved and unshared improved sanitation indicators, respectively.

⁶ The 2012 South Africa GHS does not distinguish between covered and uncovered pit latrines and therefore South Africa was excluded from the sensitivity analysis.

Table 10: Individuals not deprived in default and improved water and sanitation indicators, $t = t_2$ (per cent)

	Default water	Improved water	Default sanitation	Improved sanitation	Improved sanitation (not shared)
Benin	75	77	41	31	15
Burkina Faso	76	77	36	30	16
Cameroon	63	69	92	57	40
Chad	56	56	30	15	8
Congo, D.R.	34	51	87	41	21
Congo, Rep.	73	77	90	43	14
Côte d'Ivoire	78	78	66	47	22
Ethiopia	42	51	62	16	9
Gabon	89	93	91	63	38
Ghana	88	88	79	70	15
Guinea	75	76	83	45	21
Kenya	57	67	87	48	25
Lesotho	77	82	71	70	51
Madagascar	37	40	56	6	3
Malawi	79	79	90	14	9
Mali	66	66	88	42	24
Mozambique	53	53	61	28	24
Namibia	84	84	49	45	34
Nigeria	63	64	70	54	34
Rwanda	40	72	97	72	58
Senegal	78	78	82	63	49
Tanzania	54	56	86	34	24
Uganda	70	70	90	32	19
Zambia	63	64	84	45	28
Zimbabwe	76	77	71	62	38

Source: see Table 1.

Table 11 presents FOD and M_0 rankings using improved indicators. Despite differences in welfare levels, spatial comparisons are not vastly different and result in no major changes in the top or bottom tiers. Most countries falling in the middle move no more than a few ranks, which is not a robust result given the tight clusters of domination scores and variability introduced with bootstrapping. There are a few exceptions. Most notably, with both sets of improved indicators, Namibia and Benin perform significantly better using the FOD and AF methodologies and Rwanda performs better using the AF methodology, bringing it closer in line with FOD. Accounting for sensitivity to methodology and indicator choice, it would be reasonable to draw general conclusions regarding the relative welfare of countries, such as: ‘Senegal falls in the upper quarter’; ‘Malawi falls in the lower middle’; and ‘Gabon dominates most countries’. However, caution is pertinent in comparisons among tightly clustered

countries. For example, ‘Nigeria outperforms Rwanda’ is a claim that does not hold across indicator choice and which is not robust to differences in FOD dominance scores.

Table 11: County ranks by FOD domination scores and M_0 using the improved water and sanitation indicators ($t = t_2$)

	No distinction between shared and unshared sanitation facilities				Shared sanitation facilities are a deprivation			
	FOD dom. score	Rank	M_0	Rank	FOD dom. score	Rank	M_0	Rank
Gabon	0.16	1	0.09	1	0.17	1	0.10	1
Ghana	0.21	3	0.10	2	0.41	6	0.14	2
Congo, Rep.	0.38	7	0.21	6	0.44	9	0.23	4
Senegal	0.35	5	0.21	4	0.35	4	0.23	6
Lesotho	0.20	2	0.16	3	0.20	2	0.17	3
Zimbabwe	0.40	8	0.21	5	0.40	5	0.23	7
Cameroon	0.48	13	0.30	11	0.44	9	0.32	11
Côte d'Ivoire	0.41	9	0.24	8	0.41	7	0.28	8
Kenya	0.37	6	0.28	10	0.43	8	0.31	10
Guinea	0.53	15	0.35	15	0.52	15	0.38	15
Namibia	0.27	4	0.21	7	0.24	3	0.23	5
Zambia	0.46	10	0.33	12	0.46	13	0.35	12
Nigeria	0.48	12	0.28	9	0.46	11	0.30	9
Rwanda	0.46	11	0.34	13	0.46	11	0.36	14
Uganda	0.54	16	0.40	17	0.52	16	0.42	17
Tanzania	0.55	17	0.39	16	0.53	17	0.40	16
Malawi	0.58	18	0.47	21	0.57	18	0.48	20
Mali	0.61	19	0.44	18	0.58	19	0.47	19
Congo, D.R.	0.65	21	0.46	20	0.64	21	0.49	21
Benin	0.50	14	0.34	14	0.51	14	0.36	13
Burkina Faso	0.63	20	0.44	19	0.60	20	0.47	18
Madagascar	0.79	23	0.59	23	0.79	23	0.60	23
Mozambique	0.79	22	0.53	22	0.66	22	0.54	22
Chad	0.84	24	0.60	24	0.84	24	0.61	24
Ethiopia	0.88	25	0.65	25	0.89	25	0.66	25
Average	0.50	13	0.34	13	0.50	13	0.36	13
FOD– M_0 correlation	0.98				0.95			
Spearman rank correlation		0.97				0.97		

Source: see Table 1.

4 Conclusion

In this study, the FOD and AF approaches to evaluating multidimensional welfare were presented and applied to 26 SSA countries in two time periods. Primary data came from the highly-harmonized DHS, with the exception of the South African national household surveys and the 2001 DRC MICS. Surveys from years near 2002 and 2012 were applied for each of the 26 included countries.

Using both methodologies, we evaluated whether a country has achieved welfare gains between two time periods or whether one country dominates another. While FOD adheres to three fundamental principles of human rights, AF does not. First, FOD comparisons are independent of any applied weighting schemes and are free from assumptions regarding substitutability between included welfare indicators (indivisibility). Second, FOD cannot be established when welfare in any indicator is deteriorating, no matter how great the advancement is in other indicators (inalienability). Finally, FOD requires that domination occurs throughout the population (equality). Welfare gains among better-off groups never offset welfare losses among worse-off groups. In contrast, the AF approach relies on weighting schemes and makes use of average outcomes to compile a welfare index, and therefore violates the principles of indivisibility, inalienability, and equality. The strength of the AF approach is that it is useful in summarizing welfare trends and provides an indication of multidimensional welfare when FOD results are indeterminate.

In all analyses, we find that South Africa and/or Gabon top the country rankings. Other top performers include Ghana, the Republic of Congo, Senegal, and in some cases Zimbabwe and Lesotho. Ethiopia, Chad, Mozambique, Madagascar, Burkina Faso, and the DRC are consistently in the bottom tier. Great strides in access to basic amenities in Ethiopia and Chad have led to less differentiation among the bottom ranks. The remaining countries are tightly clustered in the middle and often difficult to differentiate with confidence. Middle-income countries outperform lower-income countries in most cases.

For the 26 SSA nations, neither AF nor FOD indicate regress, but the methodologies diverge on their assessment of progress over the study period. While the AF methodology suggests that all countries are advancing on average, the FOD, with its strict adherence to criteria that are consistent with basic human rights, is less categorical. FOD indicates that 14 countries experienced broad-based increases in welfare, while two countries show more moderate likelihoods of progress, and the remaining 10 countries neither improve nor deteriorate in terms of multidimensional welfare. For SSA as a region, we can safely conclude that welfare is improving. At the same time, challenges remain in assuring that benefits of well-documented growth are widely distributed.

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